Exercises Differential Equations

SET 1

1. For each of the following ordinary differential equations, state:
   a) Whether they are linear or non-linear,
   b) Their order,
   c) Whether they have constant or variable coefficients

Equation 1 $y^{(3)} = 6y$

Equation 2 $(y')^3 = 6y$

Equation 3 $y'' - e^x y' - y = 0$

Equation 4 $(y')^4 + 5y = 4$

Equation 5 $\sin(x)y' + 2x^2 y = \cos(x)$

Equation 6 $e^t \dfrac{dv}{dt} = kt^2$

Equation 7 $\dfrac{dv}{dt} = k\dfrac{t}{v}$

2. Verify whether or not each function $y = f(x)$ is a solution of the given differential equation.
   a) $y' + y = 0$ and $y = e^{-x}$
   b) $y'' + y = 0$ and $y = \sin(x)$
   c) $y' + y = e^{-x}$ and $y = xe^{-x}$

3. Show that the following equations define implicit solutions of the given differential equations.
   a) $e^{xy} + x + y = 0$ and $\dfrac{dy}{dx} = -\dfrac{1 + ye^{xy}}{1 + xe^{xy}}$
   b) $x^2 + y^2 = r^2$ and $\dfrac{dy}{dx} = -\dfrac{x}{y}$

SET 2

1. Solve the following first order differential equations, with separated variables. Check your work by differentiating.

a) $\dfrac{dx}{dt}=t+t^2$ b) $\dfrac{dy}{dx}=\dfrac{1}{x}$ c) $\dfrac{dz}{dx}=x\cos(x)$ d) $\dfrac{dw}{dx}=xe^{2x}$

2. Find the particular solution of the differential equation that satisfies the initial condition given:

a) $\dfrac{dy}{dx}=3x+x^2$ and $y(1)=4$ b) $\dfrac{dy}{dx}=\dfrac{-x}{\sqrt{4-x^2}}$ and $y(0)=1$

3. A function *f* is a solution of the differential equation given by $\dfrac{dy}{dx}=\dfrac{1}{x+2}-\dfrac{1}{2}\sin(x)$ for $x\geq -1$. The graph of *f* passes through the point (0, 2). Find an expression for $f(x)$.

4. Show that $y=\sin(kx)-kx\cos(kx)$, where *k* is a constant, is a solution of the first order differential equation $\dfrac{dy}{dx}=k^2x\sin(kx)$.

5. Find the solution of the differential equation $\dfrac{dy}{dx}=e^{-2x}-\dfrac{1}{x-1}, x<1$ that satisfies the initial condition $y(0)=1$.

6. A particle is projected along a straight line path. After *t* seconds, its velocity *v* metres per second is given by $v=\dfrac{1}{2+t^2}$. Find the distance travelled b the particle in the first *t* seconds.

SET 3

1. Find the particular solution of the following differential equations, giving your answer in the form $y = f(x)$ simplified as far as possible.

a) (i) $\frac{dy}{dx} = \frac{2x^2}{3y}, y = 0$ when $x = 0$

(ii) $\frac{dy}{dx} = 4xy^2, y = 1$ when $x = 0$

b) (i) $\frac{dy}{dx} = \frac{4y}{x}, y = 2$ when $x = 1$

(ii) $\frac{dy}{dx} = -3x^2y, y = 3$ when $x = 0$

2. Find the particular solution of the following differential equations. You do not need to give the expression for *y* explicitly.

a) (i) $\frac{dy}{dx} = \frac{\sin x}{\cos y}, y = 0$ when $x = \frac{\pi}{3}$

(ii) $\frac{dy}{dx} = \frac{\sec^2 x}{\sec^2 y}, y = 0$ when $x = \frac{\pi}{3}$

b) (i) $2(1+x)\frac{dy}{dx} = 1 + y^2, y = 0$ when $x = 0$

(ii) $(1+x^2)\frac{dy}{dx} = 2x\sqrt{1-y^2}, y = 0$ when $x = 0$

c) (i) $\frac{dy}{dx} = 2e^{x+2y}, y = 0$ when $x = 0$

(ii) $\frac{dy}{dx} = e^{x-y}, y = 2$ when $x = 0$

3. Find the general solution of the following differential equations, giving your answer in the form $y = f(x)$ simplified as far as possible.

a) i) $2y\frac{dy}{dx} = 3x^2$ (ii) $\frac{1}{y^2}\frac{dy}{dx} = 2x$

b) (i) $x\frac{dy}{dx} = \sec y$ (ii) $\csc x\frac{dy}{dx} = 1 + y^2$

c) (i) $(x-1)\frac{dy}{dx} = x(y+3)$ (ii) $(1-x^2)\frac{dy}{dx} = xy + y$

4. Solve the differential equation $\frac{dy}{dx} = 2y(1-x)$ given that when $x = 1, y = 1$. Give your answer in the form $y = f(x)$ simplified as far as possible.

5. Given that $\frac{dN}{dt} = -kN$, where k is a positive constant, show that $N = Ae^{-kt}$.

6. Find the general solution of the differential equation $x\frac{dy}{dx} - 4 = y^2$, giving your answer in the form $y = f(x)$.

7. Given that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ and that $y = \frac{\sqrt{3}}{2}$ when $x = \frac{1}{2}$, show that $2y = x\sqrt{k} + \sqrt{1-x^2}$ where k is a constant to be found.