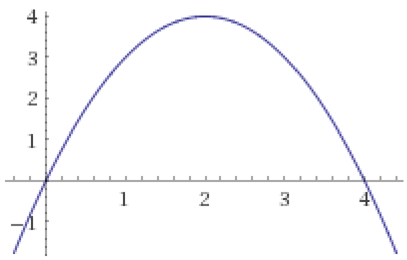
Name________________________

WARM-UP:

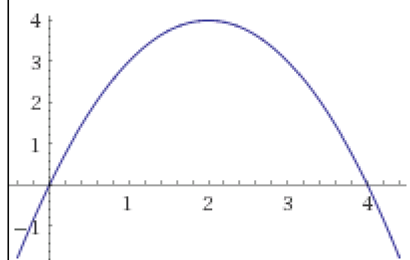
- For each graph, draw the secant line through the two points on the graph corresponding to the endpoints of the indicated interval.
- On the indicated interval, draw any tangent lines to the graph of the function that are parallel to the secant line.
- For each tangent line, estimate the x-value of the point of tangency.

1. $f(x)$ on the interval $[0, 3]$



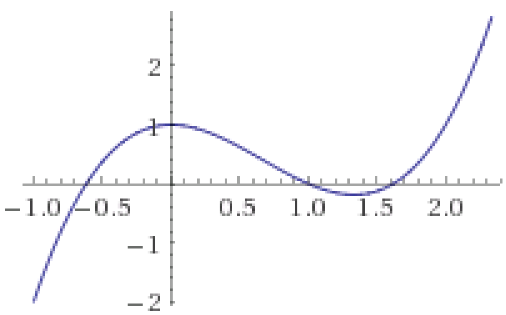
x-value(s) = ______________

2. $f(x)$ on the interval $[2, 4]$



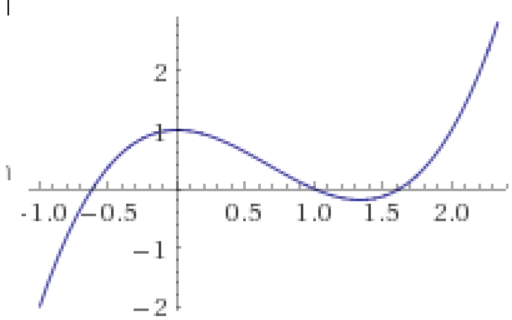
x-value(s) = ______________

3. $g(x)$ on the interval $[-1, 1]$



x-value(s) = ______________

4. $g(x)$ on the interval $[0, 2]$

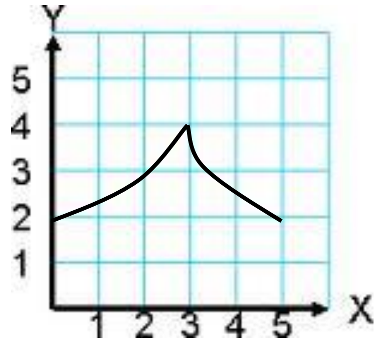


x-value(s) = ______________

Mean Value Theorem (MVT)

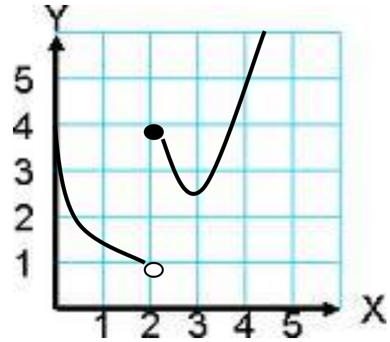
For the following graphs, draw the secant line through the two points corresponding to the endpoints of the indicated interval. Then draw any tangent lines that are parallel to this secant line.

1. $h(x)$ on the interval $[1.5, 4]$



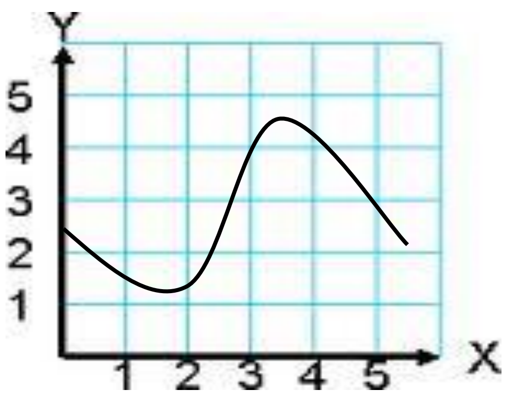
Number of tangents: ____________

2. $g(x)$ on the interval $[1, 2]$



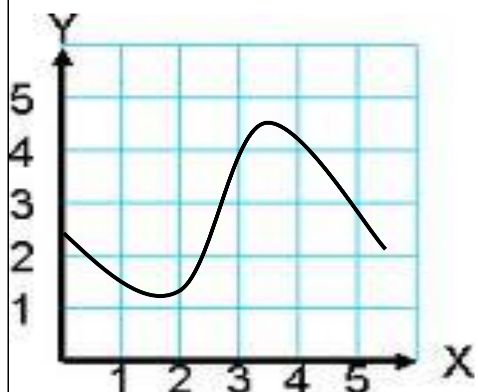
Number of tangents: ____________

3. $f(x)$ on the interval $[0, 5]$



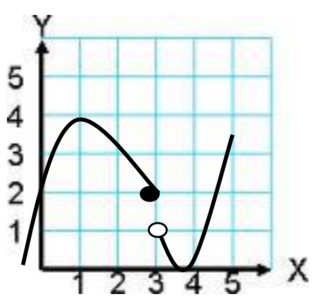
Number of tangents: ____________

4. $f(x)$ on the interval $[0, 2]$



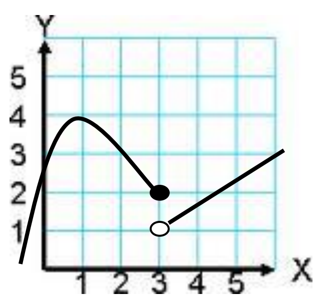
Number of tangents: ____________

5. $g(x)$ on the interval $[0, 3]$



Number of tangents: ____________

6. $g(x)$ on the interval $[2, 5]$



Number of tangents: ____________

Referring to #1-6, answer the following questions:

a) Which of the graphs are continuous on the indicated interval?

b) Which of the graphs are not continuous on the indicated interval?

c) Which of the graphs are differentiable on the open interval?

d) Which of the graphs are not differentiable on the open interval?

e) If a function is continuous on a closed interval $[a, b]$, is there a tangent line that is parallel to the secant line through the points with x-coordinates $x = a$ and $x = b$?

f) If a function is differentiable on an open interval (a, b), is there a tangent line that is parallel to the secant line through the points with x-coordinates $x = a$ and $x = b$?

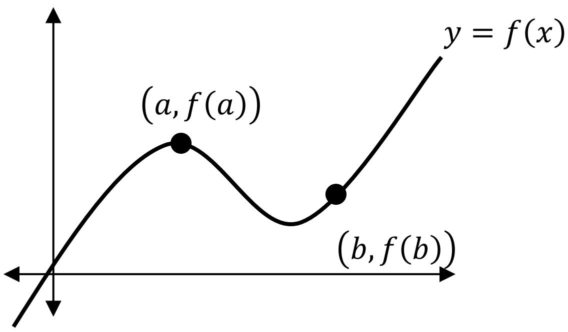
g) In the examples, in order to ensure that the graph of a function has a tangent line that is parallel to the secant line, the function must be ________________________ on the closed interval and _______________________ on the open interval. For which of these functions of these functions do these two conditions hold?

The **<u>Mean Value Theorem</u>** is one of the most important theoretical tools in Calculus. On the AP Calculus AB Exam, you not only need to know the theorem, but will be expected to apply it to a variety of situations.

Mean Value Theorem: If f is ____________________ on the ________________ interval $[a, b]$ and _______________________ on the ____________ interval (a, b), then there exists at least one number c in the interval (a, b) such that:

Alternate form: There is at least one x-value in (a, b) at which the ________________________ rate of change of f is equal to the ____________________ rate of change of f on $[a, b]$.

Geometric Interpretation:



1. $\dfrac{f(b) - f(a)}{b - a}$ is the ________________________ of the line segment joining the points $(a, f(a))$ and $(b, f(b))$.
2. $f'(c)$ is the _____________________ of the line _______________________ to the graph of f at the point ________________.
3. On the graph of f, locate the x-value, $x = c$, that is ensured by the Mean Value Theorem. Mark this x-value on the above graph.

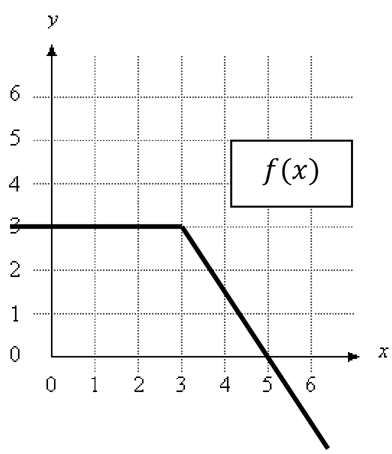
For each function, determine if the Mean Value Theorem applies. If it does apply, explain what conclusions you can draw from it. If it does not apply, state why not.

1. $g(x) = 4x^3 - x^2 + 4$ on the interval $[-1, 1]$.

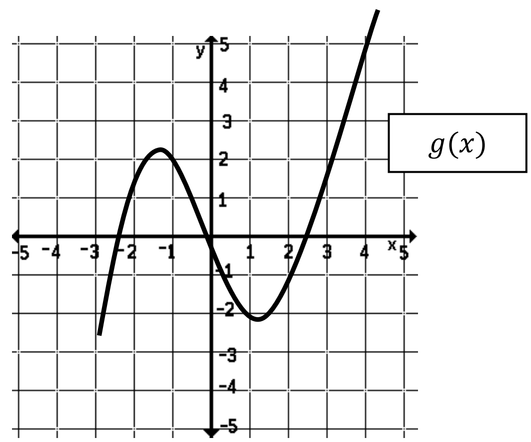
2. $f(x) = \dfrac{1}{x-1}$ on the interval $[0, 3]$.

3. $h(x) = \dfrac{1}{x+1}$ on the interval $[0, 3]$.

4. $f(x)$ on the interval $[0, 5]$



5. $g(x)$ on the interval $[-2, 2]$



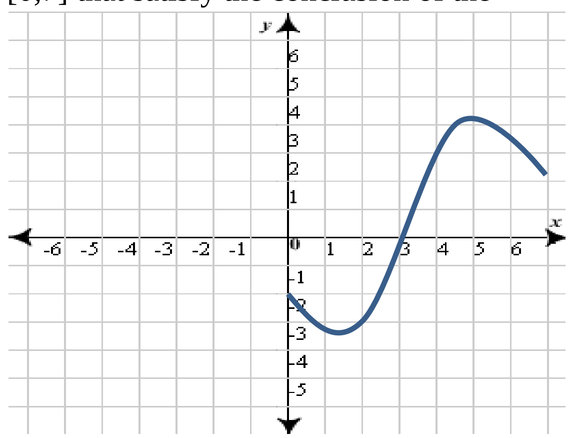
EXAMPLE 1: If the function f is defined on $[1, 3]$ by $f(x) = 4 - \frac{3}{x}$, show that the Mean Value Theorem can be applied to f and find a number c which satisfies the conclusion.

EXAMPLE 2: A cyclist rides on a straight road with positive velocity $v(t)$, in miles per minute, and time t minutes, where v is a differentiable function of t. Selected values $v(t)$ are given in the table below for $0 \le t \le 30$. Is there a time t, $0 < t < 30$, when the cyclist has a negative acceleration? Justify your answer.

t	0	5	10	15	20	25	30
$v(t)$	0.2	0.4	0.6	0.5	0.5	0.4	0.3

PRACTICE:

1. Use the graph of f to estimate the numbers in [0,7] that satisfy the conclusion of the Mean Value Theorem.



2. Determine whether $f(x) = 3x^2 + x - 4$ satisfies the MVT on the interval $[1,5]$. If it does, find all numbers c, in (1,5) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

3. Determine whether $f(x) = \cos x - \sin x$ satisfies the MVT on the interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ If it does, find all numbers c, in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

4. Suppose that $s(t) = t^2 - t + 4$ is the position function of the motion of a particle moving in a straight line.
   a) Explain why the function s satisfies the hypothesis of the MVT.

   b) Find the value of t in [0,3] where the instantaneous velocity is equal to the average velocity.

5. The differentiable function R gives the rate at which fuel flows into a tank over time, where the rate $R(t)$ is measured in gallons per hour and time t is measured in hours, $0 \leq t \leq 12$. The table below shows rates measured every two hours for the given 12-hour period. Is there some time t, $0 < t < 12$ for which $R'(t) = 0$? Justify your answer.

t	$R(t)$
0	78
2	84
4	88
6	92
8	94
10	93
12	88