C4 DIFFERENTIAL EQUATIONS Worksheet A

1 Find the general solution of each differential equation.

a $\dfrac{dy}{dx} = (x+2)^3$ **b** $\dfrac{dy}{dx} = 4\cos 2x$ **c** $\dfrac{dx}{dt} = 3e^{2t} + 2$

d $(2-x)\dfrac{dy}{dx} = 1$ **e** $\dfrac{dN}{dt} = t\sqrt{t^2+1}$ **f** $\dfrac{dy}{dx} = xe^x$

2 Find the particular solution of each differential equation.

a $\dfrac{dy}{dx} = e^{-x}$, $y = 3$ when $x = 0$ **b** $\dfrac{dy}{dt} = \tan^3 t \sec^2 t$, $y = 1$ when $t = \frac{\pi}{3}$

c $(x^2-3)\dfrac{du}{dx} = 4x$, $u = 5$ when $x = 2$ **d** $\dfrac{dy}{dx} = 3\cos^2 x$, $y = \pi$ when $x = \frac{\pi}{2}$

3 **a** Express $\dfrac{x-8}{x^2-x-6}$ in partial fractions.

b Given that

$$(x^2 - x - 6)\frac{dy}{dx} = x - 8,$$

and that $y = \ln 9$ when $x = 1$, show that when $x = 2$, the value of y is $\ln 32$.

4 Find the general solution of each differential equation.

a $\dfrac{dy}{dx} = 2y + 3$ **b** $\dfrac{dy}{dx} = \sin^2 2y$ **c** $\dfrac{dy}{dx} = xy$

d $(x+1)\dfrac{dy}{dx} = y$ **e** $\dfrac{dy}{dx} = \dfrac{x^2-2}{y}$ **f** $\dfrac{dy}{dx} = 2\cos x\cos^2 y$

g $\sqrt{x}\,\dfrac{dy}{dx} = e^{y-3}$ **h** $y\dfrac{dy}{dx} = xy^2 + 3x$ **i** $\dfrac{dy}{dx} = xy\sin x$

j $\dfrac{dy}{dx} = e^{2x-y}$ **k** $(y-3)\dfrac{dy}{dx} = xy(y-1)$ **l** $\dfrac{dy}{dx} = y^2\ln x$

5 Find the particular solution of each differential equation.

a $\dfrac{dy}{dx} = \dfrac{x}{2y}$, $y = 3$ when $x = 4$ **b** $\dfrac{dy}{dx} = (y+1)^3$, $y = 0$ when $x = 2$

c $(\tan^2 x)\dfrac{dy}{dx} = y$, $y = 1$ when $x = \frac{\pi}{2}$ **d** $\dfrac{dy}{dx} = \dfrac{y+2}{x-1}$, $y = 6$ when $x = 3$

e $\dfrac{dy}{dx} = x^2\tan y$, $y = \frac{\pi}{6}$ when $x = 0$ **f** $\dfrac{dy}{dx} = \sqrt{\dfrac{y}{x+3}}$, $y = 16$ when $x = 1$

g $e^x\dfrac{dy}{dx} = x\,\mathrm{cosec}\, y$, $y = \pi$ when $x = -1$ **h** $\dfrac{dy}{dx} = \dfrac{1+\cos y}{2x^2\sin y}$, $y = \frac{\pi}{3}$ when $x = 1$

6 A quantity has the value N at time t hours and is increasing at a rate proportional to N.

a Write down a differential equation relating N and t.

b By solving your differential equation, show that

$$N = A\mathrm{e}^{kt},$$

where A and k are constants and k is positive.

Given that when $t = 0$, $N = 40$ and that when $t = 5$, $N = 60$,

c find the values of A and k,

d find the value of N when $t = 12$.

7 At time $t = 0$, a piece of radioactive material has mass 24 g. Its mass after t days is m grams and is decreasing at a rate proportional to m.

a By forming and solving a suitable differential equation, show that

$$m = 24\mathrm{e}^{-kt},$$

where k is a positive constant.

After 20 days, the mass of the material is found to be 22.6 g.

b Find the value of k.

c Find the rate at which the mass is decreasing after 20 days.

d Find how long it takes for the mass of the material to be halved.

8 A quantity has the value P at time t seconds and is decreasing at a rate proportional to $\sqrt{P}$.

a Write down a differential equation relating P and t.

b By solving your differential equation, show that

$$P = (a - bt)^2,$$

where a and b are constants.

Given that when $t = 0$, $P = 400$,

c find the value of a.

Given also that when $t = 30$, $P = 100$,

d find the value of P when $t = 50$.

9 **a** Express $\frac{1}{(1+x)(1-x)}$ in partial fractions.

In an industrial process, the mass of a chemical, m kg, produced after t hours is modelled by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = k\mathrm{e}^{-t}(1 + m)(1 - m),$$

where k is a positive constant.

Given that when $t = 0$, $m = 0$ and that the initial rate at which the chemical is produced is 0.5 kg per hour,

b find the value of k,

c show that, for $0 \le m < 1$, $\ln\left(\frac{1+m}{1-m}\right) = 1 - \mathrm{e}^{-t}$.

d find the time taken to produce 0.1 kg of the chemical,

e show that however long the process is allowed to run, the maximum amount of the chemical that will be produced is about 462 g.