C4 DIFFERENTIAL EQUATIONS *Worksheet B*

1 **a** Find the general solution of the differential equation

$$\frac{dy}{dx} = xy^3.$$ **(3)**

b Given also that $y = \frac{1}{2}$ when $x = 1$, find the particular solution of the differential equation, giving your answer in the form $y^2 = f(x)$. **(3)**

2 Given that $y = \frac{\pi}{4}$ when $x = 1$, solve the differential equation

$$\frac{dy}{dx} = x \sec y \operatorname{cosec}^3 y.$$ **(6)**

3 The rate of growth in the number of yeast cells, N, present in a culture after t hours is proportional to N.

a By forming and solving a differential equation, show that

$$N = Ae^{kt},$$

where A and k are positive constants. **(3)**

Initially there are 200 yeast cells in the culture and after 2 hours there are 3000 yeast cells in the culture. Find, to the nearest minute, after how long

b there are 10 000 yeast cells in the culture, **(5)**

c the number of yeast cells is increasing at the rate of 5 per second. **(4)**

4 **a** Find $\int x \ln x \, dx$. **(4)**

b Given that $y = 4$ when $x = 2$, solve the differential equation

$$\frac{dy}{dx} = xy \ln x, \quad x > 0, \quad y > 0,$$

and hence, find the exact value of y when $x = 1$. **(4)**

5 Given that $y = 0$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^{x+y} \cos x.$$ **(8)**

6 The temperature in a room is 10°C. A heater is used to raise the temperature in the room to 25°C and then turned off. The amount by which the temperature in the room exceeds 10°C is θ°C, at time t minutes after the heater is turned off.

It is assumed that the rate at which θ decreases is proportional to θ.

a By forming and solving a suitable differential equation, show that

$$\theta = 15e^{-kt},$$

where k is a positive constant. **(5)**

Given that after half an hour the temperature in the room is 20°C,

b find the value of k. **(3)**

The heater is set to turn on again if the temperature in the room falls to 15°C.

c Find how long it takes before the heater is turned on. **(2)**

7 In an experiment to investigate the formation of ice on a body of water, a thin circular disc of ice is placed on the surface of a tank of water and the surrounding air temperature is kept constant at −5°C.

In a model of the situation, it is assumed that the disc of ice remains circular and that its area, A cm^2 after t minutes, increases at a rate proportional to its perimeter.

a Show that

$$\frac{dA}{dt} = k\sqrt{A},$$

where k is a positive constant. **(3)**

b Show that the general solution of this differential equation is

$$A = (pt + q)^2,$$

where p and q are constants. **(3)**

Given that when $t = 0$, $A = 25$ and that when $t = 20$, $A = 40$,

c find how long it takes for the area to increase to 50 cm^2. **(4)**

8 **a** Express $\frac{x+4}{(1+x)(2-x)}$ in partial fractions. **(3)**

b Solve the differential equation

$$\frac{dy}{dx} = \frac{y(x+4)}{(1+x)(2-x)},$$

given that $y = 2$ when $x = 3$. **(5)**

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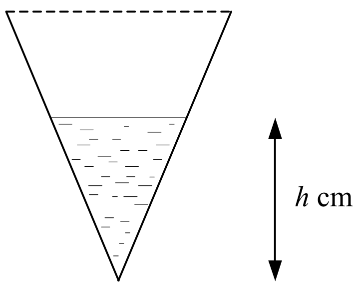

The diagram shows a container in the shape of a right-circular cone. A quantity of water is poured into the container but this then leaks out from a small hole at its vertex.

In a model of the situation it is assumed that the rate at which the volume of water in the container, V cm^3, decreases is proportional to V. Given that the depth of the water is h cm at time t minutes,

a show that

$$\frac{dh}{dt} = -kh,$$

where k is a positive constant. **(5)**

Given also that $h = 12$ when $t = 0$ and that $h = 10$ when $t = 20$,

b show that

$$h = 12e^{-kt},$$

and find the value of k, **(5)**

c find the value of t when $h = 6$. **(2)**