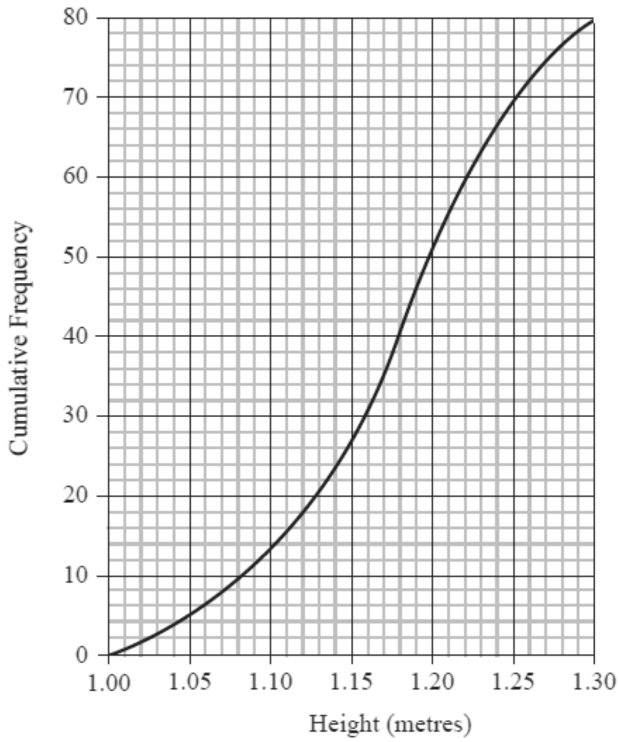
1. The heights in metres of a random sample of 80 boys in a certain age group were measured and the following cumulative frequency graph obtained.



(a) (i) Estimate the median of these data.

(ii) Estimate the interquartile range for these data.

(3)

(b) (i) Produce a frequency table for these data, using a class width of 0.05 metres.

(ii) Calculate unbiased estimates of the mean and variance of the heights of the population of boys in this age group.

(5)

(c) A boy is selected at random from these 80 boys.

(i) Find the probability that his height is less than or equal to 1.15 metres.

(ii) Given that his height is less than or equal to 1.15 metres, find the probability that his height is less than or equal to 1.12 metres.

(5)
(Total 13 marks)

2. (a) Consider the set of numbers a, $2a$, $3a$, ..., na where a and n are positive integers.

(i) Show that the expression for the mean of this set is $\dfrac{a(n+1)}{2}$.

(ii) Let $a = 4$. Find the minimum value of n for which the sum of these numbers exceeds its mean by more than 100.

(6)

(b) Consider now the set of numbers $x_1, \ldots, x_m, y_1, \ldots, y_1, \ldots, y_n$ where $x_i = 0$ for $i = 1, \ldots, m$ and $y_i = 1$ for $i = 1, \ldots, n$.

(i) Show that the mean M of this set is given by $\dfrac{n}{m+n}$ and the standard deviation S by $\dfrac{\sqrt{mn}}{m+n}$.

(ii) Given that $M = S$, find the value of the median.

(11)
(Total 17 marks)

3. At a nursing college, 80 % of incoming students are female. College records show that 70 % of the incoming females graduate and 90 % of the incoming males graduate.
A student who graduates is selected at random. Find the probability that the student is male, giving your answer as a fraction in its lowest terms.

(Total 5 marks)

4. (a) A box of biscuits is considered to be underweight if it weighs less than 228 grams. It is known that the weights of these boxes of biscuits are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams. What is the probability that a box is underweight?

(2)

(b) The manufacturer decides that the probability of a box being underweight should be reduced to 0.002.

(i) Bill's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.

(ii) Sarah's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation.

(6)

(c) After the probability of a box being underweight has been reduced to 0.002, a group of customers buys 100 boxes of biscuits. Find the probability that at least two of the boxes are underweight.

(3)
(Total 11 marks)

5. The ten numbers $x_1, x_2, ..., x_{10}$ have a mean of 10 and a standard deviation of 3.

Find the value of $\sum_{i=1}^{10}(x_i - 12)^2$.

(Total 6 marks)

6. The weight loss, in kilograms, of people using the slimming regime *SLIM3M* for a period of three months is modelled by a random variable X. Experimental data showed that 67 % of the individuals using *SLIM3M* lost up to five kilograms and 12.4 % lost at least seven kilograms. Assuming that X follows a normal distribution, find the expected weight loss of a person who follows the *SLIM3M* regime for three months.

(Total 5 marks)

7. After being sprayed with a weedkiller, the survival time of weeds in a field is normally distributed with a mean of 15 days.

(a) If the probability of survival after 21 days is 0.2, find the standard deviation of the survival time.

(3)

When another field is sprayed, the survival time of weeds is normally distributed with a mean of 18 days.

(b) If the standard deviation of the survival time is unchanged, find the probability of survival after 21 days.

(2)
(Total 5 marks)

8. The distance travelled by students to attend Gauss College is modelled by a normal distribution with mean 6 km and standard deviation 1.5 km.

(a) (i) Find the probability that the distance travelled to Gauss College by a randomly selected student is between 4.8 km and 7.5 km.

(ii) 15% of students travel less than d km to attend Gauss College. Find the value of d.

(7)

At Euler College, the distance travelled by students to attend their school is modelled by a normal distribution with mean μ km and standard deviation σ km.

(b) If 10% of students travel more than 8 km and 5% of students travel less than 2 km, find the value of μ and of σ.

(6)

The number of telephone calls, T, received by Euler College each minute can be modelled by a Poisson distribution with a mean of 3.5.

(c) (i) Find the probability that at least three telephone calls are received by Euler College in **each** of two successive one-minute intervals.

(ii) Find the probability that Euler College receives 15 telephone calls during a randomly selected five-minute interval.

(8)
(Total 21 marks)

9. The lifts in the office buildings of a small city have occasional breakdowns. The breakdowns at any given time are independent of one another and can be modelled using a Poisson Distribution with mean 0.2 per day.

(a) Determine the probability that there will be exactly four breakdowns during the month of June (June has 30 days).

(3)

(b) Determine the probability that there are more than three breakdowns during the month of June.

(2)

(c) Determine the probability that there are no breakdowns during the first five days of June.

(2)

(d) Find the probability that the first breakdown in June occurs on June 3^{rd}.

(3)

(e) It costs 1850 euros to service the lifts when they have breakdowns. Find the expected cost of servicing lifts for the month of June.

(1)

(f) Determine the probability that there will be no breakdowns in exactly 4 out of the first 5 days in June.

(2)
(Total 13 marks)

10. After a shop opens at 09:00 the number of customers arriving in any interval of duration t minutes follows a Poisson distribution with mean $\frac{t}{10}$.

(a) (i) Find the probability that exactly five customers arrive before 10:00.

(ii) Given that exactly five customers arrive before 10:00, find the probability that exactly two customers arrive before 09:30.

(7)

(b) Let the second customer arrive at T minutes after 09:00.

(i) Show that, for $t > 0$,

$$\mathrm{P}(T > t) = \left(1+\frac{t}{10}\right)\mathrm{e}^{-\frac{t}{10}}.$$

(ii) Hence find in simplified form the probability density function of T.

(iii) Evaluate $\mathrm{E}(T)$.
(You may assume that, for $n \in \mathbb{Z}^+$ and $a > 0$, $\lim\limits_{t\to\infty} t^n \mathrm{e}^{-at} = 0$.)

(12)
(Total 19 marks)

11. Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

(a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212.

(1)

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water.

(3)

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water is greater than 0.99.

(4)

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B.

(6)

(e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X, follows a Poisson distribution such that $P(X = 2) = P(X = 3) + P(X = 4)$.

(i) Find the value of m.

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period.

(6)
(Total 20 marks)

12. The random variable X follows a Poisson distribution with mean m and satisfies

$$P(X = 1) + P(X = 3) = P(X = 0) + P(X = 2).$$

(a) Find the value of m correct to four decimal places.

(4)

(b) For this value of m, calculate $P(1 \le X \le 2)$.

(2)
(Total 6 marks)